Parallel Shortest Lattice Vector Enumeration on Graphics Cards

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Introduction

Lattices: crash course

GPUs

GPU-Enum Algorithm

The Future

Why GPU?

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(Source: MSI)

Why GPU?

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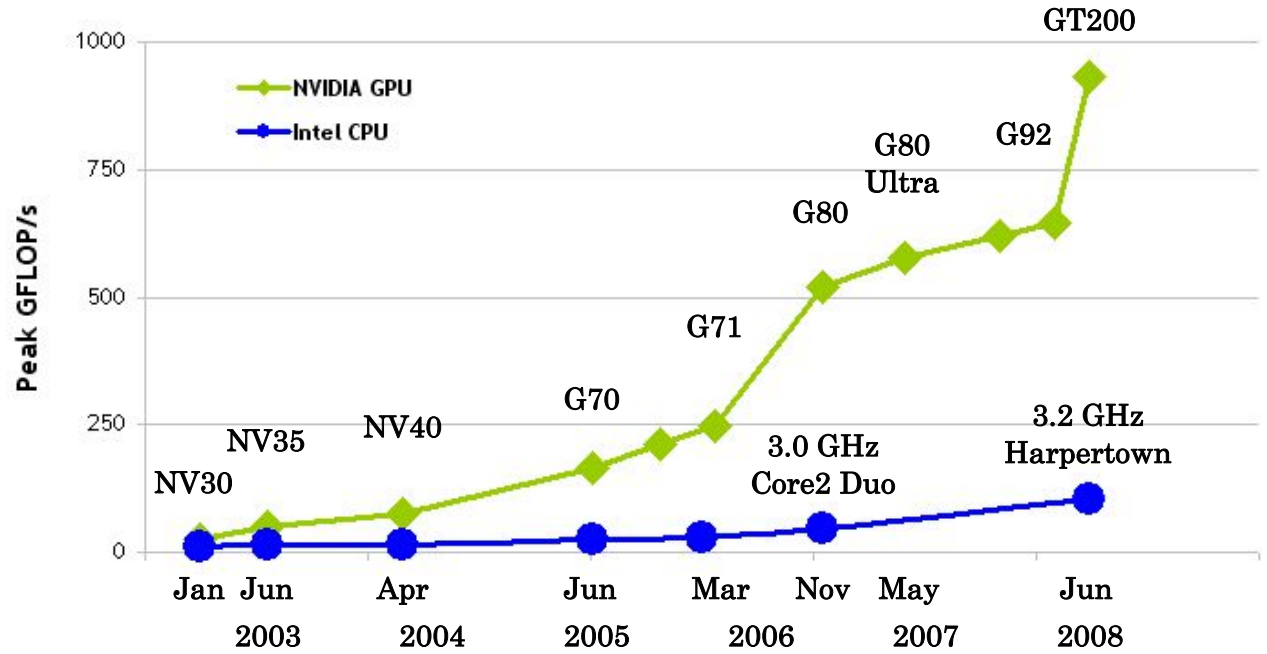

GT200 = GeForce GTX 280	G71 = GeForce 7900 GTX	NV35 = GeForce FX 5950 Ultra
G92 = GeForce 9800 GTX	G70 = GeForce 7800 GTX	NV30 = GeForce FX 5800
G80 = GeForce 8800 GTX	NV40 = GeForce 6800 Ultra	

(Source: NVIDIA)

Why GPU?

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Absolute transistor count:

AMD Athlon 64 X2	CPU	154 mio.
Intel Core 2 Duo	CPU	291 mio.
Intel Core 2 Quad	CPU	582 mio.
NVIDIA G8800 GTX	GPU	680 mio.
Indel Core i7-980X Extreme	CPU	1.2 mrd.
ATI RV870	GPU	2.2 mrd.
NVIDIA Fermi	GPU	3.0 mrd.

CUDA framework

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Warning: Sales Talk

Your own personal supercomputer for $<$ €500.

Nvidia CUDA Framework:

- Run ‘general’ programs on GPU
- More complex operations, data types, branching...
- Recent GPU required
- Theory: 1TFlop (practice: 200 GFlop)

Crypto on GPU

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Current Crypto Applications

- Ciphers:
  - RSA[a], ECC[b], AES[c]
- Cryptanalysis:
  - Factoring[d]
  - Brute force

[a] Moss, Page, Smart / Szerwinski, Guneysu / Fleissner
[b] Szerwinski, Guneysu
[c] Manavski / Harrison, Waldron
[d] Bernstein, Chen, Cheng, Lange, Yang

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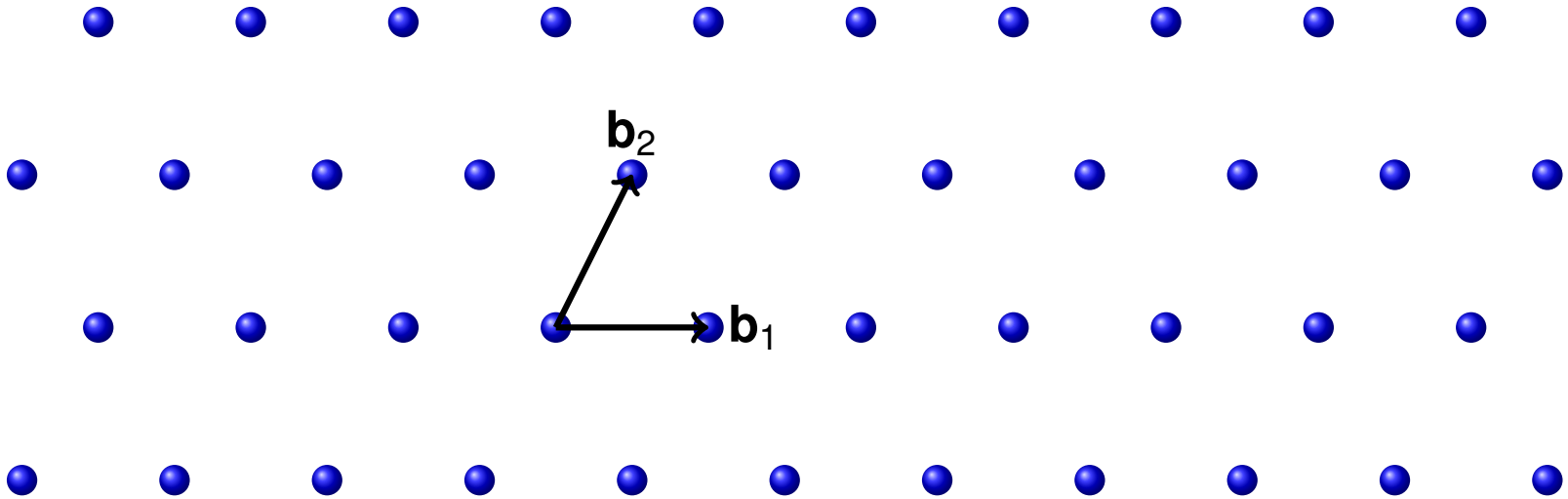

- Basis matrix $\mathbf{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ with $\mathbf{b}_i \in \mathbb{R}^d$
- Lattice: $\mathcal{L}(\mathbf{B}) = \{\sum_{i=1}^{n} x_i \mathbf{b}_i, x_i \in \mathbb{Z}\}$

Shortest Vector Problem (SVP)

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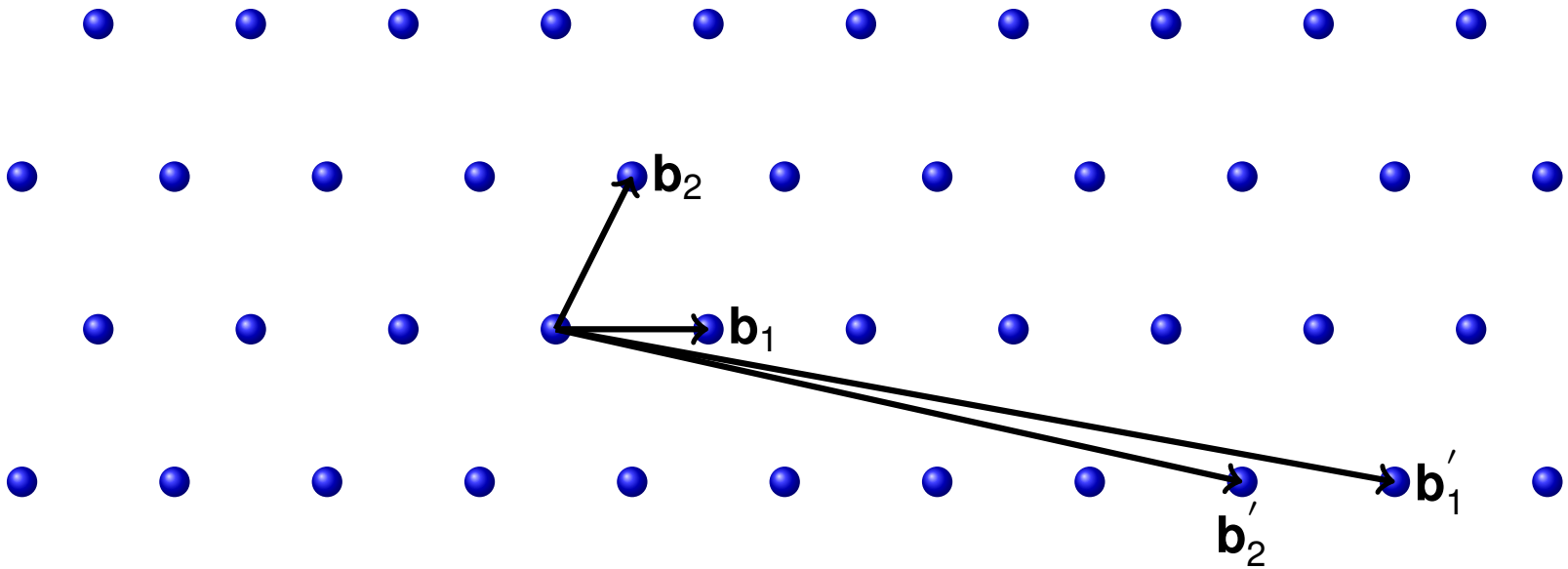

- Basis not unique
- Idea: 'good' basis $\mathbf{B}$ and 'bad' basis $\mathbf{B}'$
- Finding a shortest vector is hard with $\mathbf{B}'$

Shortest Vector Problem (SVP)

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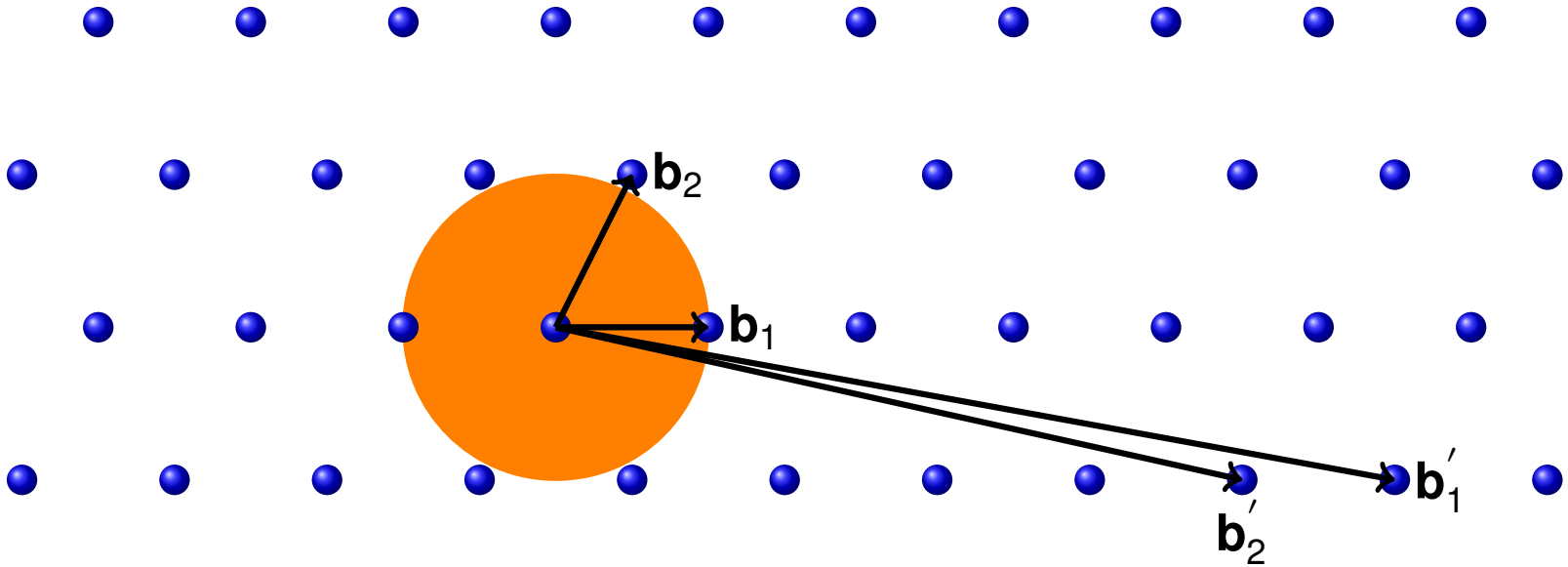

- Basis not unique
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- Finding a shortest vector is hard with **B**′

Algorithms for SVP

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Shortest Vector Problem

$$\min_{\mathbf{x} \in \mathbb{Z}^n} \left\| \mathbf{Bx} \right\|_2$$

SVP Algorithms:

LLL	approximate solution, polynomial time
BKZ	better approximation
...	
Enum	exact solution, exponential time
Sieving	exact solution, exponential time, probabilistic

$\Longrightarrow$ This talk: focus on Enum

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Processor

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Nvidia GTX280:

- 240 cores, scalar processors
- 30 multiprocessors (8 cores each)
- 1.3 GHz
- 1GB Global Memory
- 32 & 64-bit integers, single + double precision FP

Programming model

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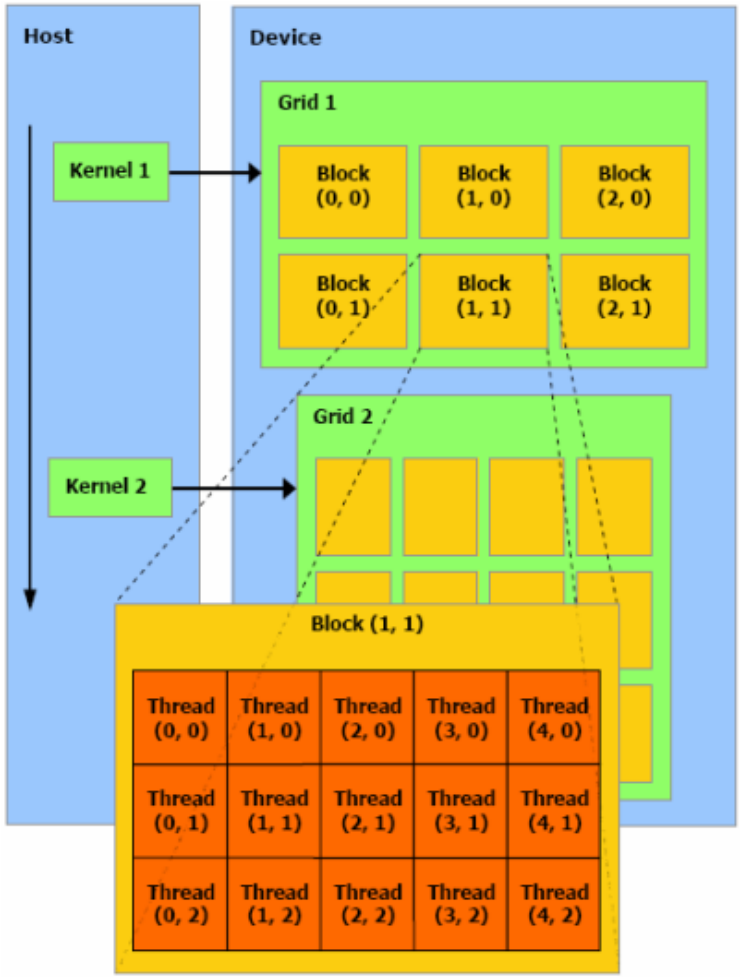

(Source: CUDA programming guide)

Memory types

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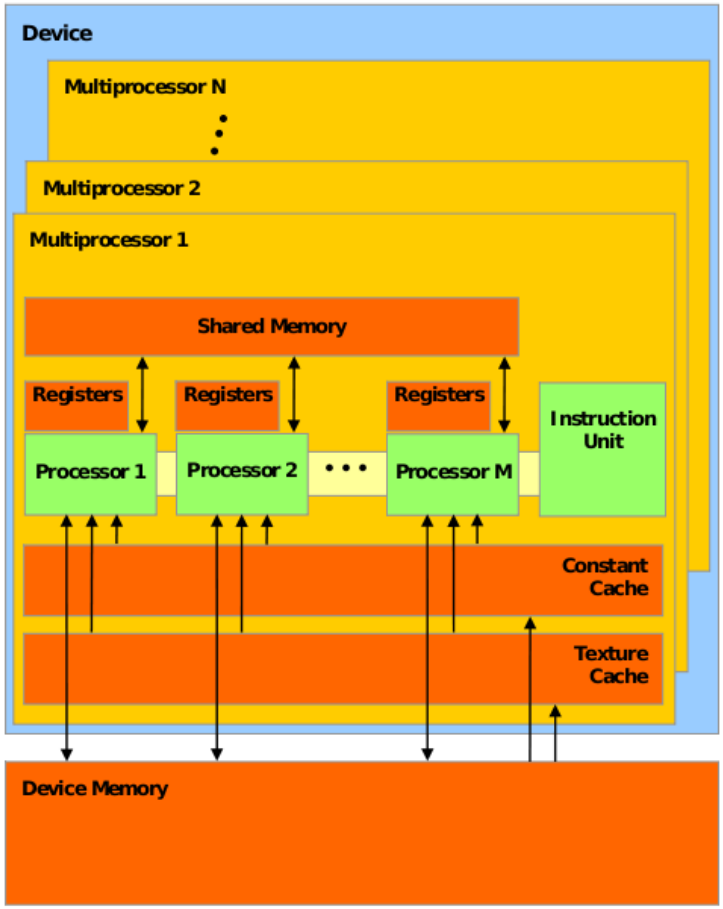

(Source: CUDA programming guide)

Code Example

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```
CUDA_SAFE_CALL(cudaMalloc((void **)&d_A, sizeof(norminttype)));
CUDA_SAFE_CALL(cudaMemcpy(d_A,h_A,sizeof(norminttype),cudaMemcpyHostToDevic
CUDA_SAFE_CALL(cudaFree(d_A));
gpuenum <<<16,32>>>(paramlist);
```

Code Example

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gpuenum <<<16,32>>>(paramlist);

__global__ void gpuenum(paramlist)
    for (int k = 0; k < alpha + 1; k++) {
        fptype cintsumtmp = 0.0;
        for (int j = alpha; j <= beta; j++) {
            cintsumtmp += (x[j + sharedOffset] * MU(j, k));
        }
    }
    if (startlevel >= alpha && l > localA)
        continue;
```

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Enumeration

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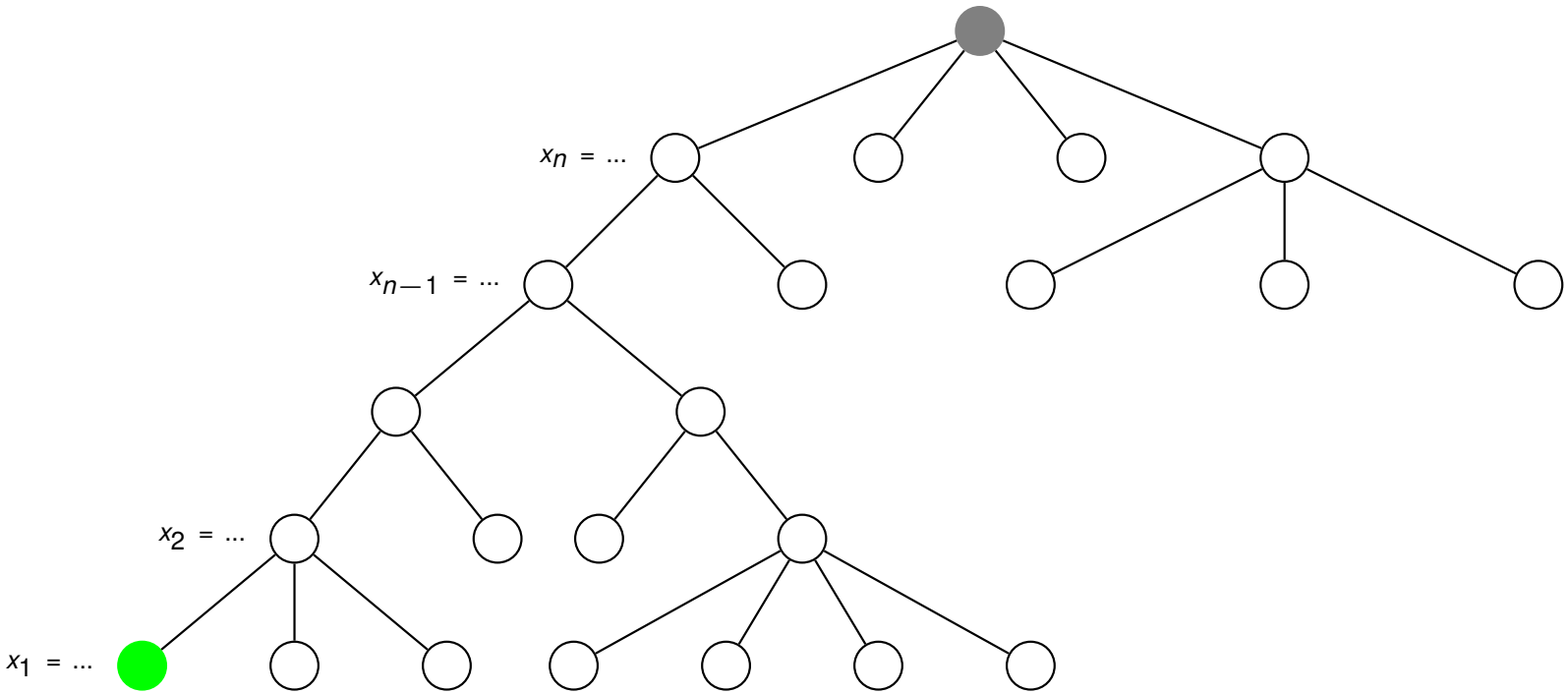

$\mathbf{x} = [1, 0, \ldots, 0] \quad \longrightarrow \quad \text{optimum } A = \|\mathbf{Bx}\|_2^2$

Enumeration

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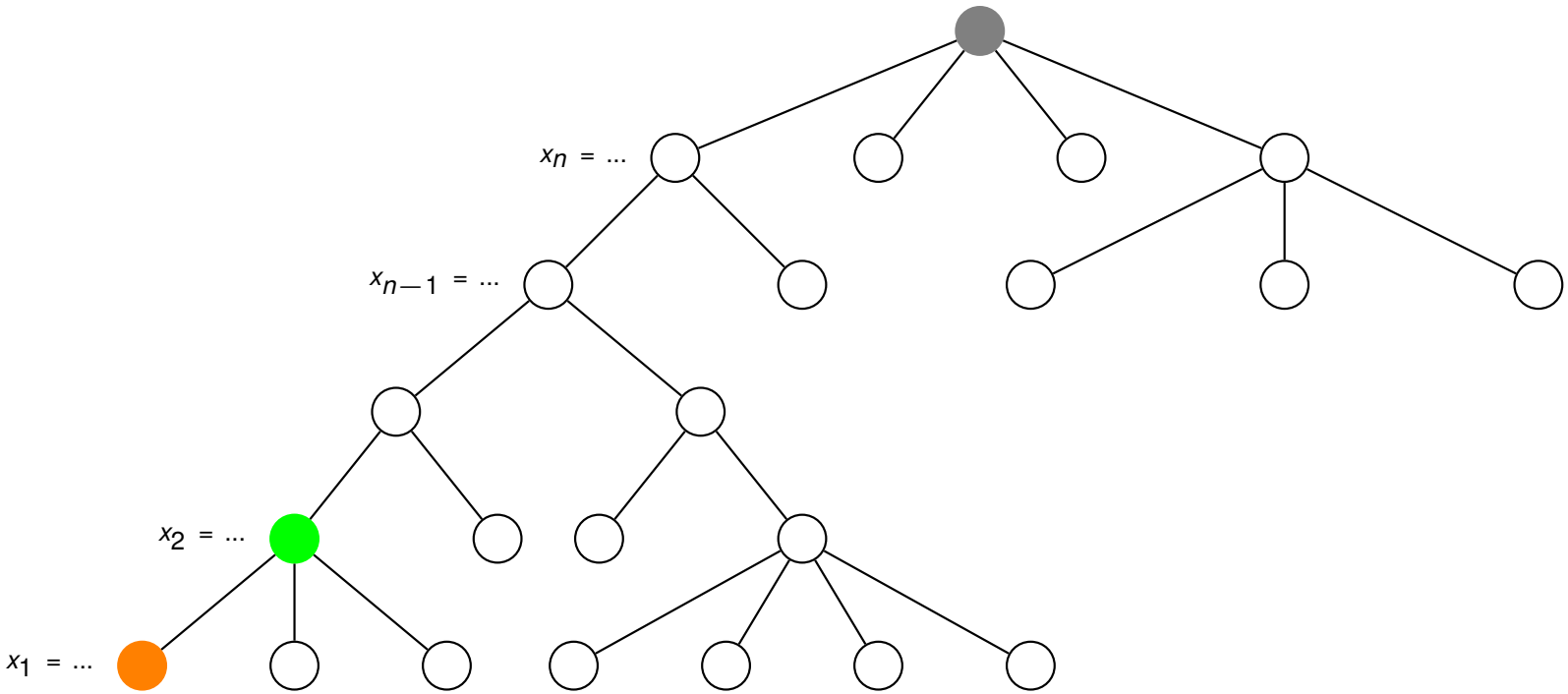

Intermediate norm l_2 of vector $[\star, x_2, x_3, \ldots, x_n]$
with $l_i \geq l_{i+1}$

Enumeration

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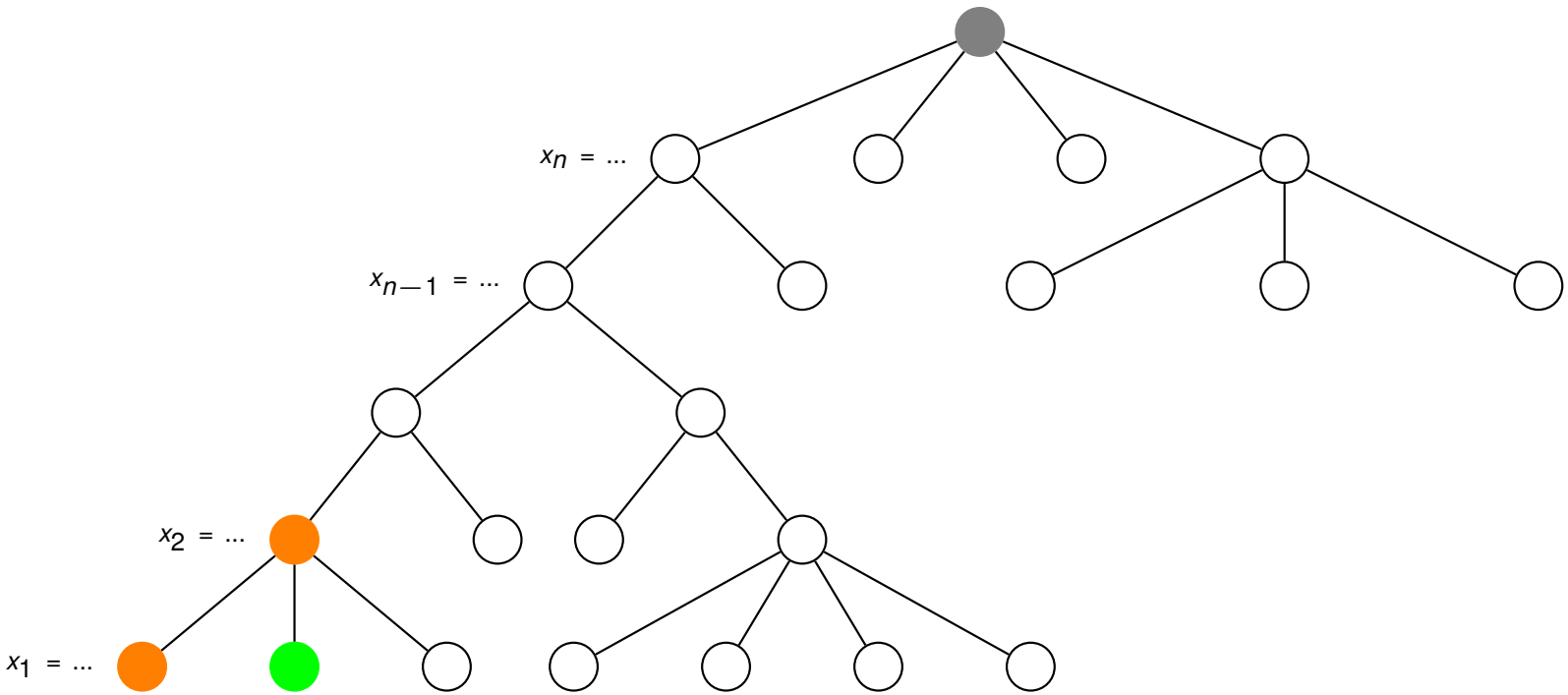

New optimum $A = \|\mathbf{Bx}\|_2^2$

Enumeration

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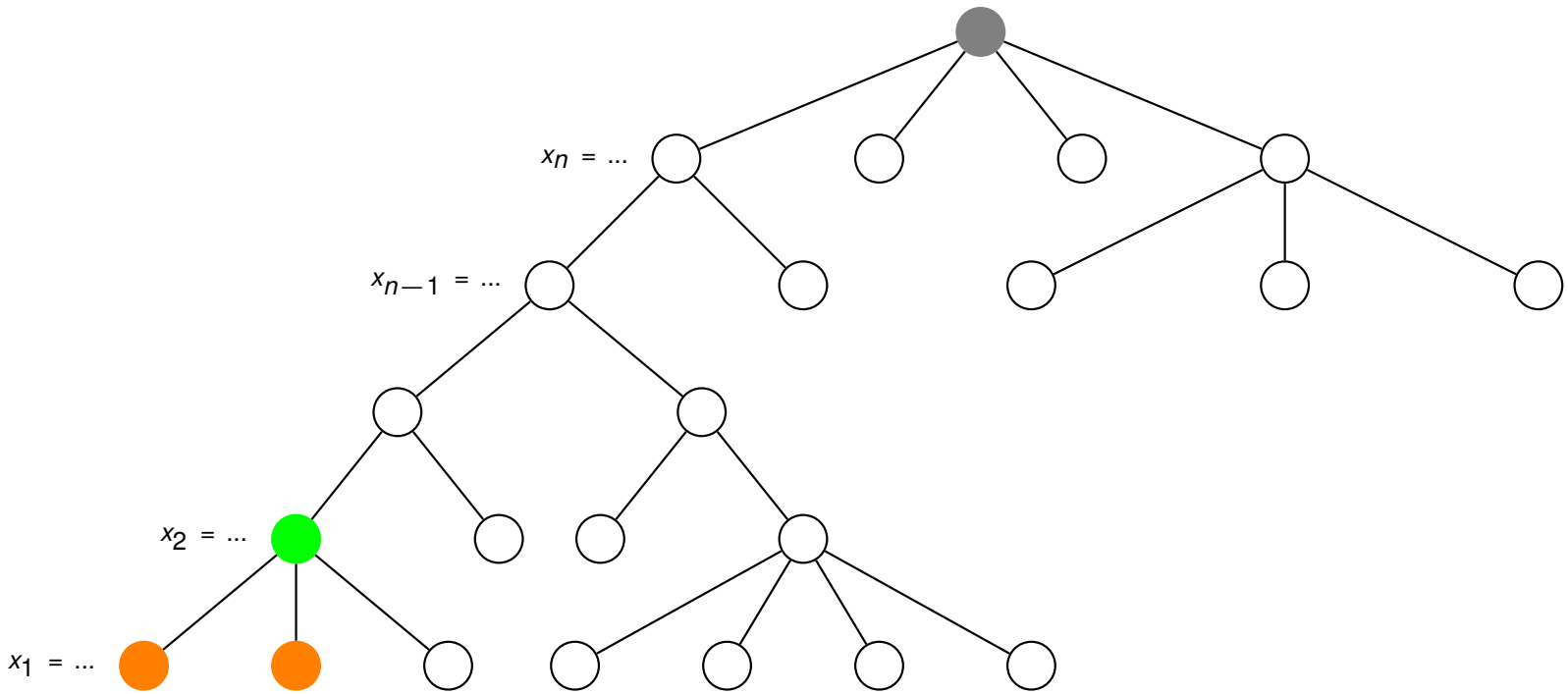

Cut off branch if $l_i > A$.

Enumeration

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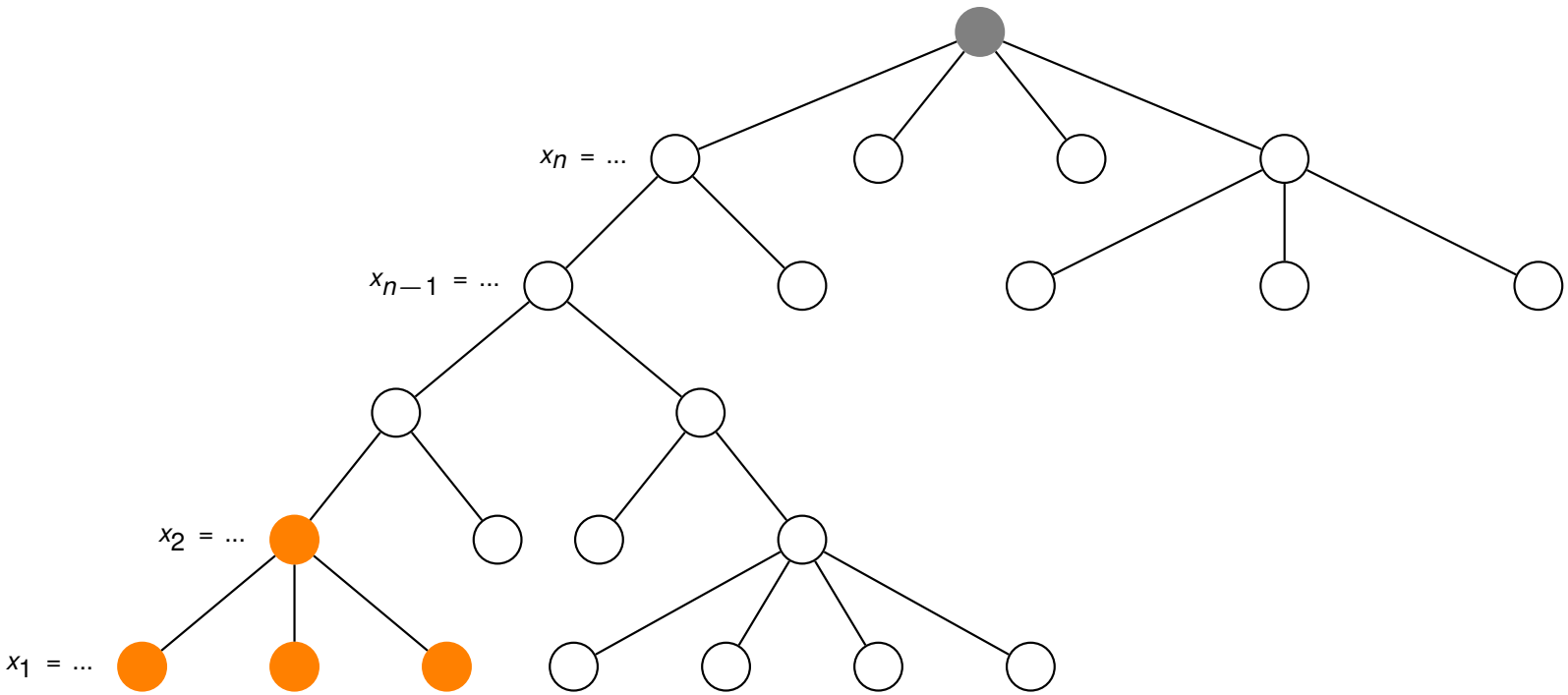

Enumeration

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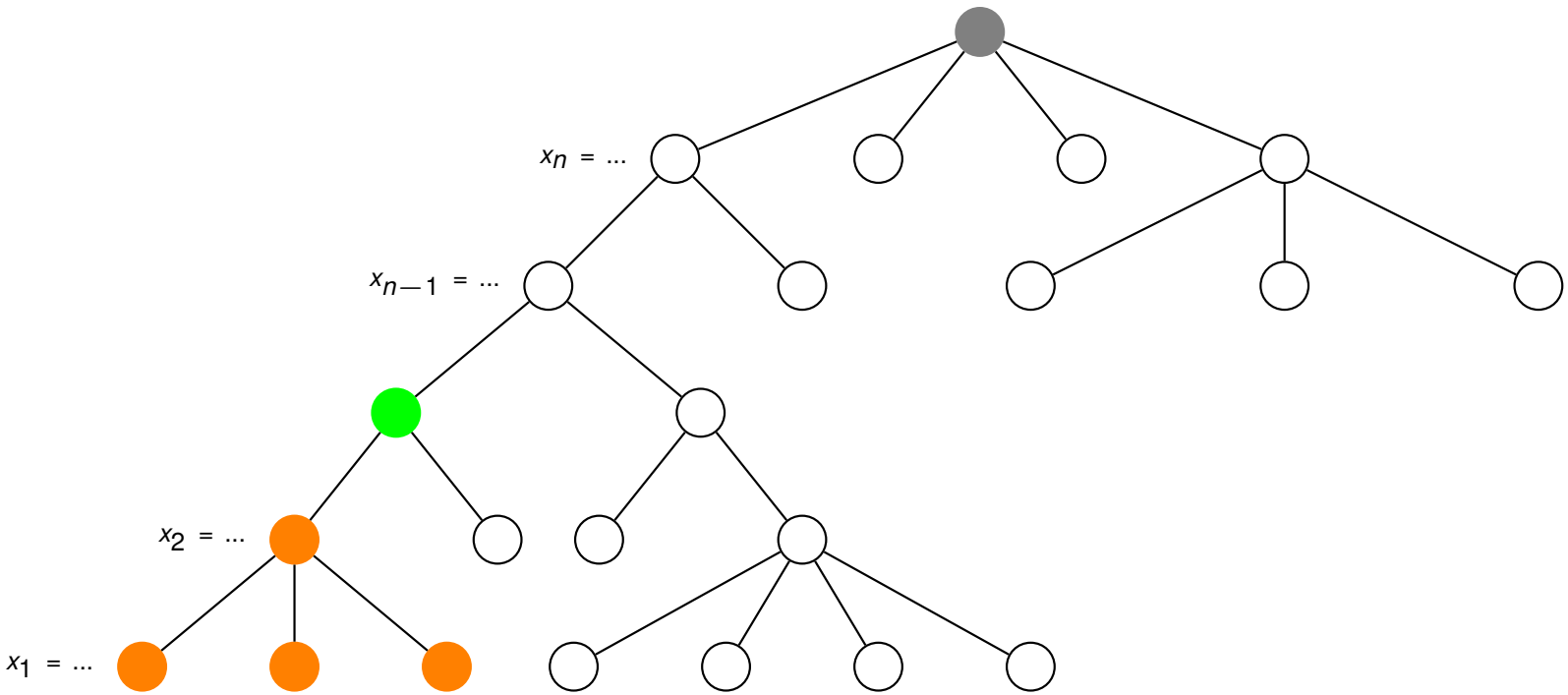

Enumeration

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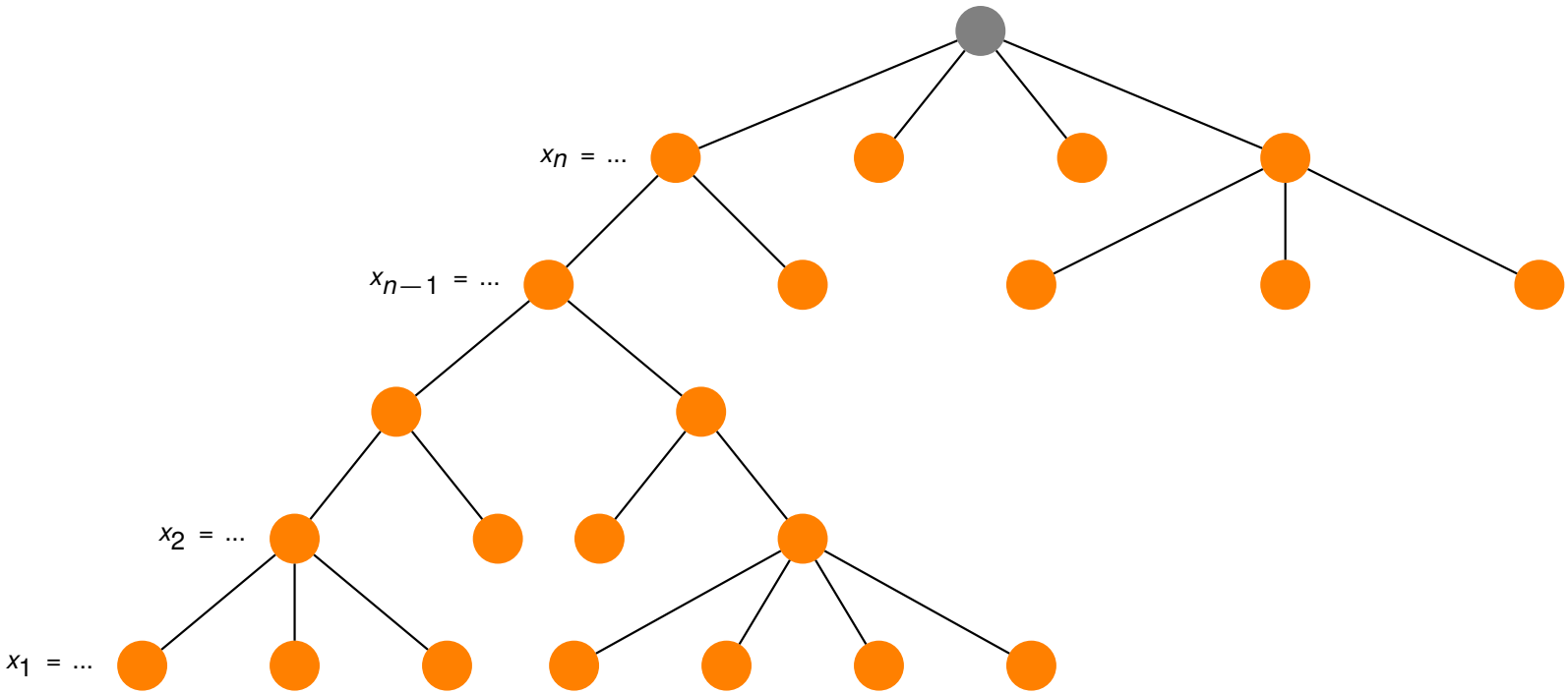

Algorithm Flow

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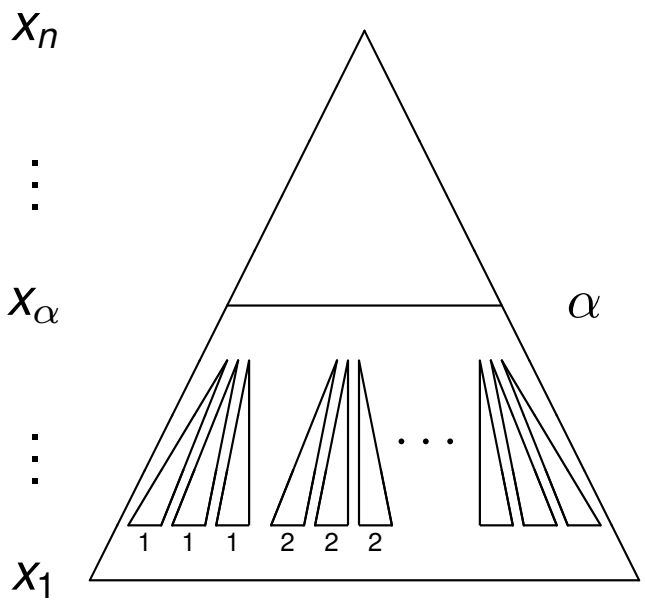
How to find independent subtrees?



Basic idea

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Input: $\mathbf{B}$, A, α, n

Compute the Gram-Schmidt decomposition of $\mathbf{b}_i$

Output: $(x_1, \ldots, x_n)$ with $\left\| \sum_{i=1}^{n} x_i \mathbf{b}_i \right\|$ minimal

Basic idea

TECHNISCHE UNIVERSITÄT DARMSTADT

Input: $\mathbf{B}$, A, α, n

Compute the Gram-Schmidt decomposition of $\mathbf{b}_i$
CPU: generate $\mathbf{x}_i = [0, \ldots, 0, x_\alpha, \ldots, x_n]$

Output: $(x_1, \ldots, x_n)$ with $\left\| \sum_{i=1}^{n} x_i \mathbf{b}_i \right\|$ minimal

Basic idea

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GPU thread: run a sub-enum on $\mathbf{x}_i$, if new optimum $\longrightarrow$ store in $\mathbf{x}$

Output: $(x_1, \ldots, x_n)$ with $\left\| \sum_{i=1}^{n} x_i \mathbf{b}_i \right\|$ minimal

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$\Longrightarrow$ horrible performance 

Thread execution time

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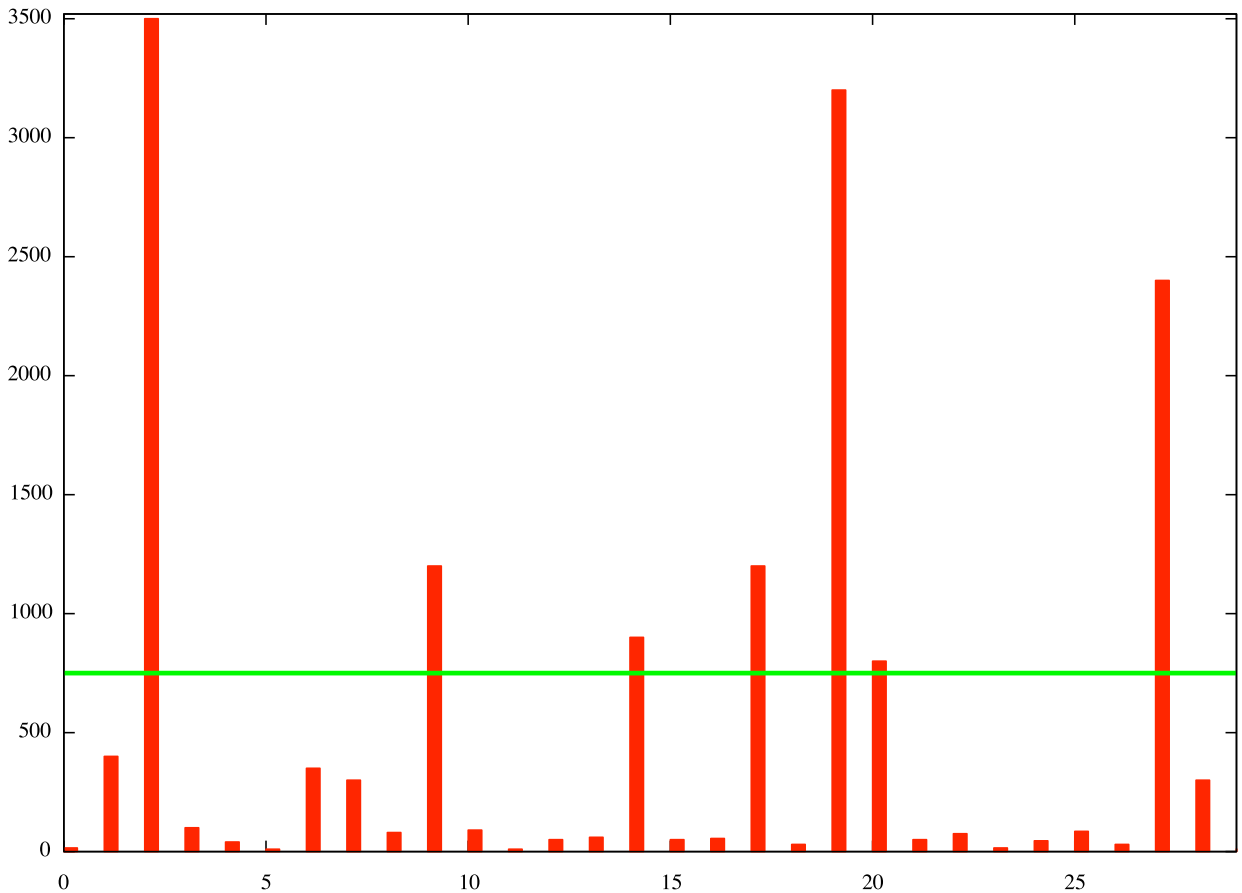

GPU Enumeration

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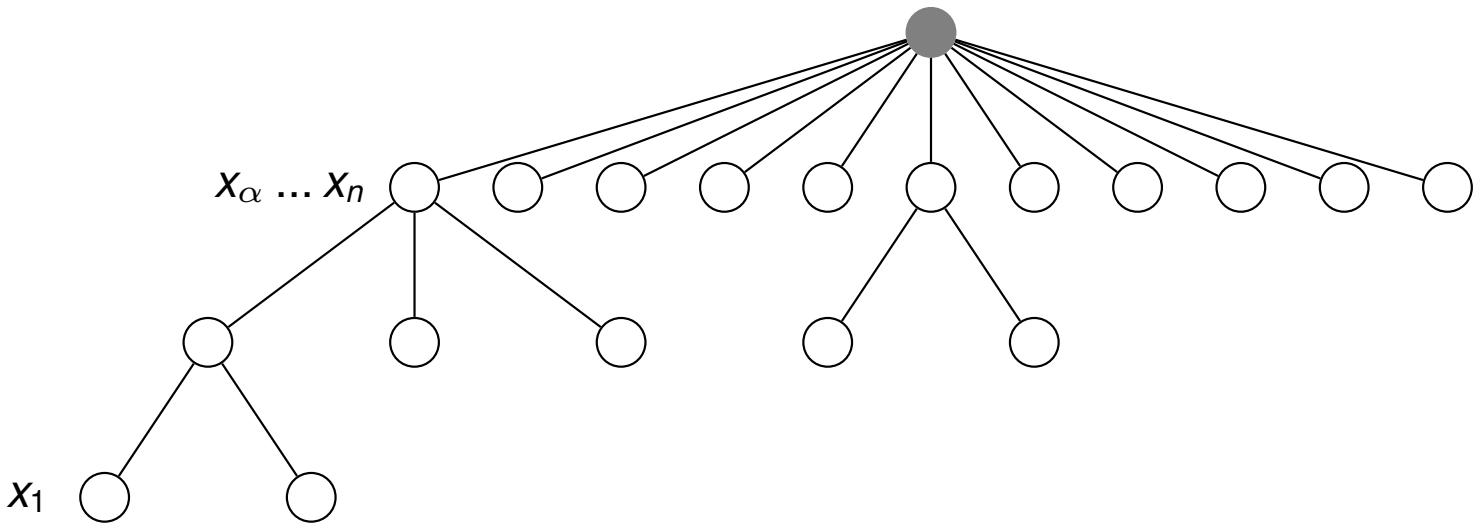

GPU Enumeration

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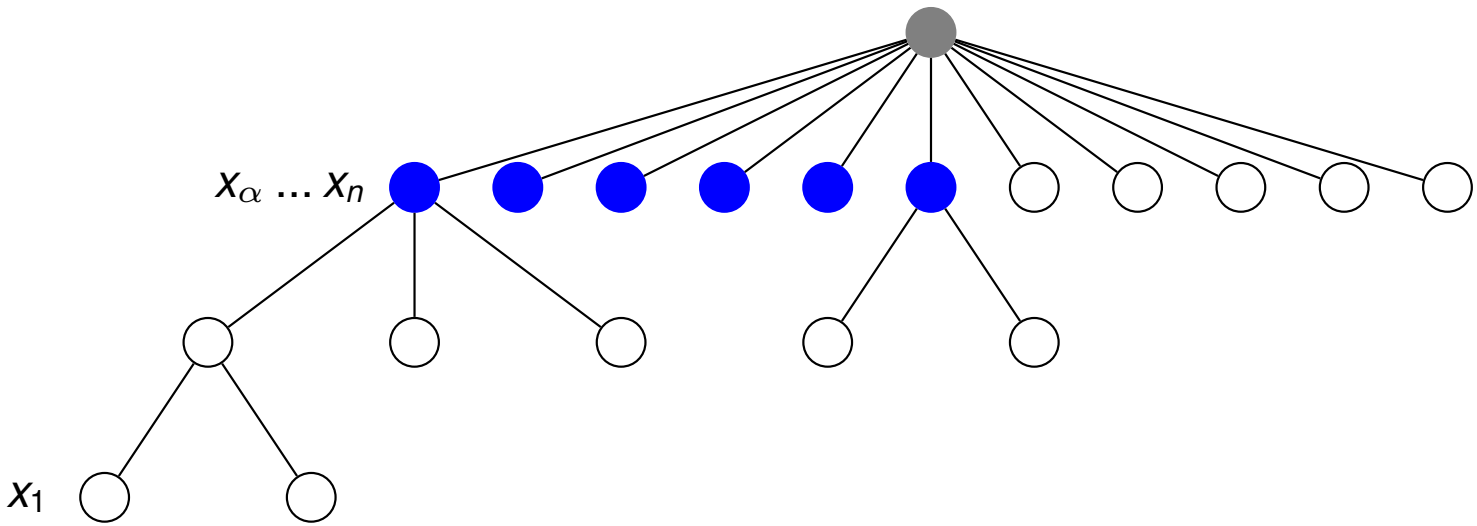

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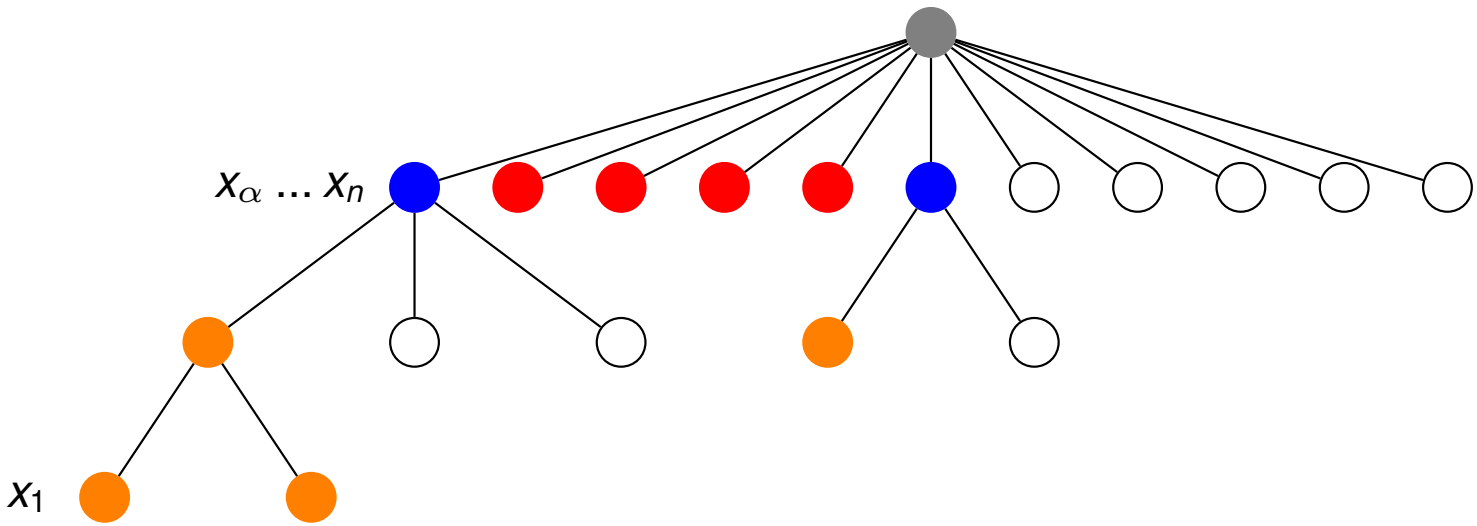

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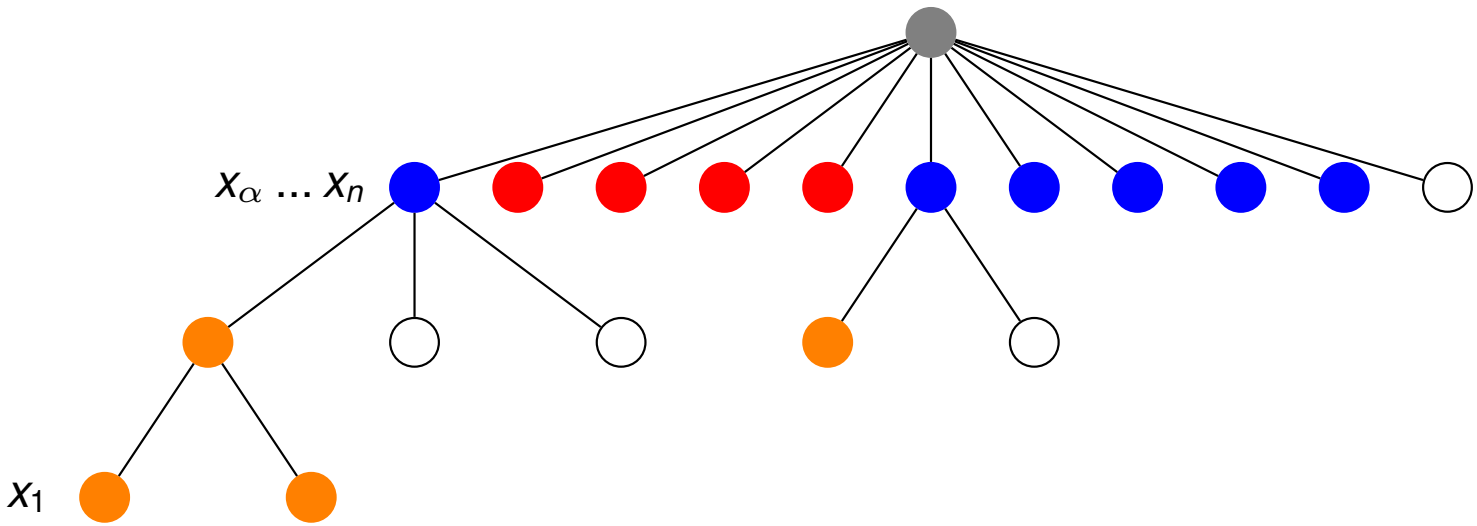

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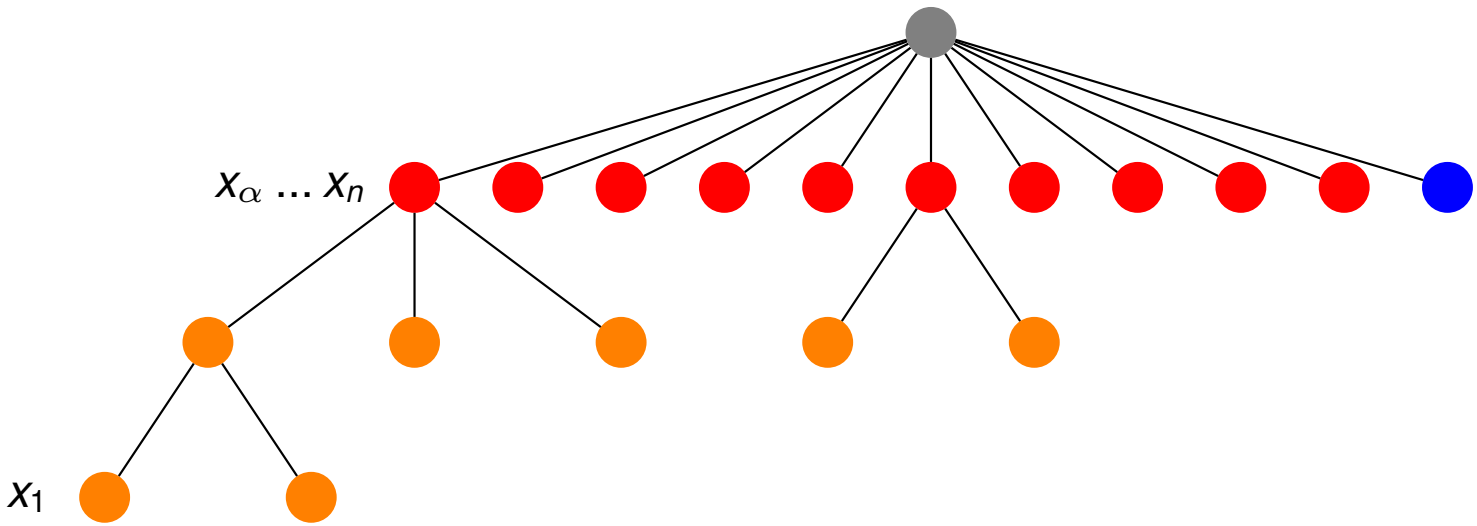

Iterating

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Input: $\mathbf{B}$, A, α, n

Compute the Gram-Schmidt decomposition of $\mathbf{b}_i$

while *true* **do**

CPU: generate some $\mathbf{x}_i = [0, \ldots, 0, x_\alpha, \ldots, x_n]$

CPU: Get enum state $\bar{\mathbf{x}}_i = [\bar{x}_1, \ldots, \bar{x}_{\alpha-1}, x_\alpha, \ldots, x_n]$

end

Output: $(x_1, \ldots, x_n)$ with $\left\| \sum_{i=1}^{n} x_i \mathbf{b}_i \right\|$ minimal

Iterating

TECHNISCHE UNIVERSITÄT DARMSTADT

Input: $\mathbf{B}$, A, α, n

Compute the Gram-Schmidt decomposition of $\mathbf{b}_i$
while *true* **do**
  CPU: generate some $\mathbf{x}_i = [0, \ldots, 0, x_\alpha, \ldots, x_n]$
  GPU thread:
  while *there are* $\mathbf{x}_i$ *left* **do**
    Start enum for a certain $\mathbf{x}_i$ or continue enum for $\bar{\mathbf{x}}_i$
    Stop enum after $\mathbf{M}$ steps, store the state $\{l_i, \bar{\mathbf{x}}_i, s_i = \mathbf{M}\}$
  end
  CPU: Get enum state $\bar{\mathbf{x}}_i = [\bar{x}_1, \ldots, \bar{x}_{\alpha-1}, x_\alpha, \ldots, x_n]$
end

Output: $(x_1, \ldots, x_n)$ with $\left\| \sum_{i=1}^{n} x_i \mathbf{b}_i \right\|$ minimal

Implementation details

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Some facts & figures:

- Dimension 50, 100,000 starting vectors $\rightarrow$ upload & download $\sim$ 20 MB of data to GPU
- CPU top enum: very fast (low dimension)
- GPU runs for $>$ 10 seconds per iteration, iteration overhead is limited
- Share new optimal values among GPU threads

Throughput

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Throughput:

- CPU: $25 \cdot 10^6$ steps/s
- GPU: $200 \cdot 10^6$ steps/s

Throughput on GPU depends on:

- Lattice dimension n
- Length of sub-enumerations
- Number of parallel threads, uploaded points...

Results

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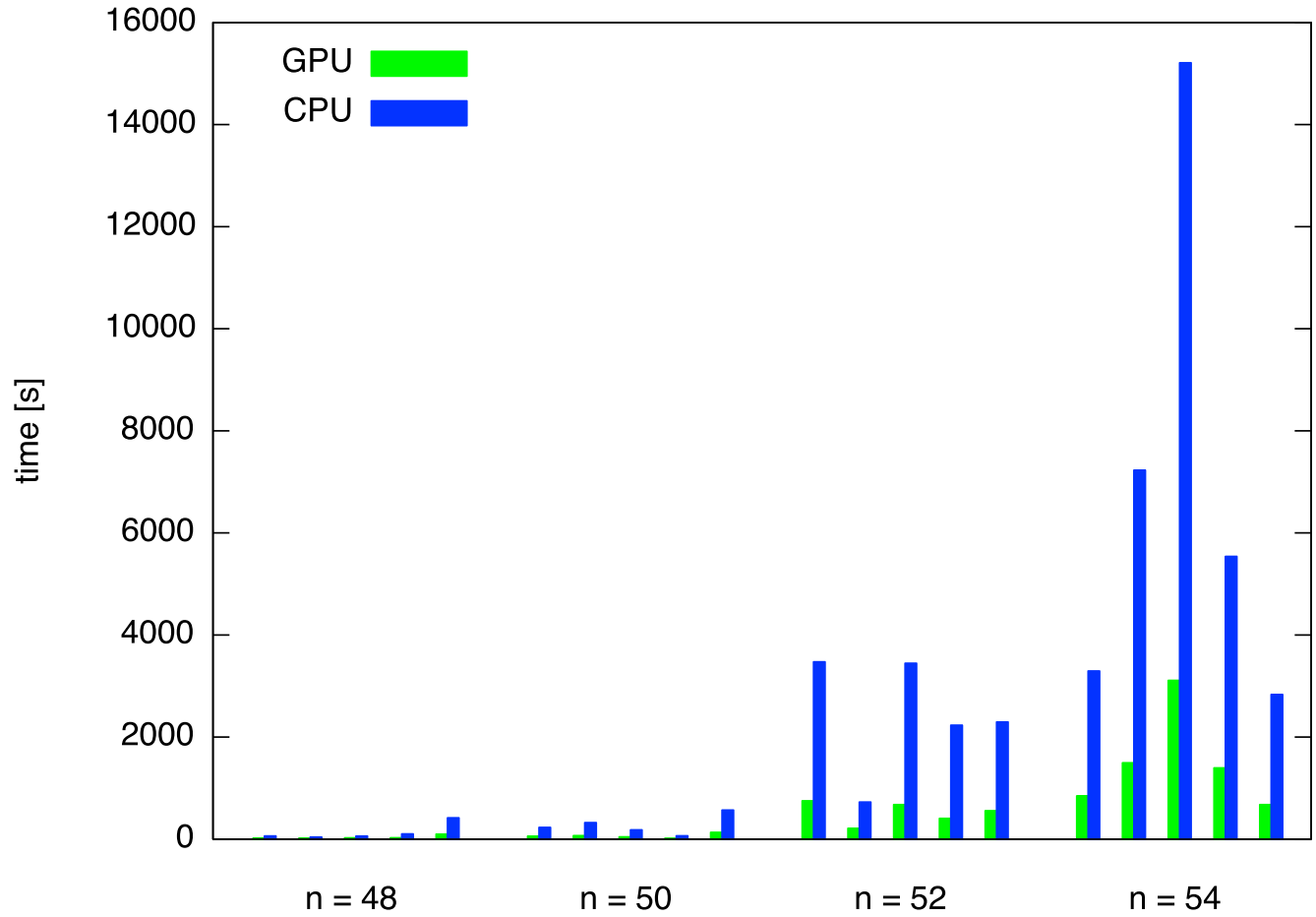

Results

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n	44	48	52	56	60
`fpLLL` - ENUM	17.7s	136s	2434s	137489s	-
CUDA - ENUM	11.7s	37.0s	520s	30752s	274268s
	66%	27%	21%	22%	-

Speedup in this model: ≈ 5

`http://homes.esat.kuleuven.be/~jhermans/gpuenum/index.html`

Results

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CPU machine

- 4-core CPU
- $900
- speedup 4
- total cost 91 · t

GPU machine

- 4-core CPU + 2 x GTX295 GPU
- $2200
- speedup 24
- total cost 225 · t

Speedup in this model: 2.4

Ideas for the Future

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- Generalize ideas (not specific for gpu's... clusters?)
- Use full power of CPU (now: idle during gpu-time)
- Test on NVIDIA Fermi cards

(... and we will climb the highest mountains)

Finally...

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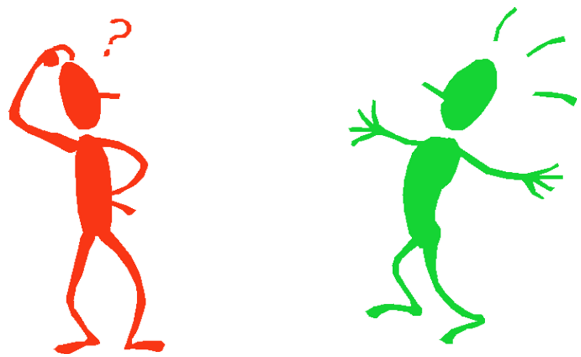

Questions?

Problems

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- GTX 280 can make 13 times more fmadds than Core2Extreme CPU
- using CUDA profiler
- diverging branches, warp serializations, ...
- lots of ALUs left idle